63_3_4

403508

ASD-TDR-63-267

CATALOGED BY ASTIA
AS AD No.

ASPECTS OF RELIABILITY UNDER CONDITIONS OF ELEVATED TEMPERATURE CREEP AND FATIGUE

TECHNICAL DOCUMENTARY REPORT NO. ASD-TDR-63-267

MARCH 1963

DIRECTORATE OF MATERIALS AND PROCESSES
AERONAUTICAL SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

403 508

Project No. 7351, Task No. 735106

DDC
RECEIVED
MAY 14 1963
TISIA D

(Prepared under Contract No. AF 33(616)-6288
by Ohio State University Research Foundation, Columbus, Ohio;
A. M. Freudenthal, Author.)

NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

Qualified requesters may obtain copies of this report from the Armed Services Technical Information Agency, (ASTIA), Arlington Hall Station, Arlington 12, Virginia.

This report has been released to the Office of Technical Services, U.S. Department of Commerce, Washington 25, D.C., in stock quantities for sale to the general public.

Copies of this report should not be returned to the Aeronautical Systems Division unless return is required by security considerations, contractual obligations, or notice on a specific document.

27-455C, 800, 5-8-63

Foreword

This report was prepared by Dr. A. M. Freudenthal, New York, N. Y. under USAF Contract AF 33(616)-6288. The contract was initiated under Project No. 7351, "Metallic Materials," Task No.735106 "Behavior of Metals." The contract was administered by the Ohio State University Research Foundation. The work was monitored by the Directorate of Materials and Processes, Aeronautical Systems Division, under the direction of Mr. W. J. Trapp.

This report covers work performed during the period February 1 to October 1, 1962.

Abstract

The solution of the problem of attaining adequate safety and reliability in supersonic aircraft structures operating under conditions under which the damaging effects of cycle sensitivity (fatigue) and time-sensitivity (creep) of the structural material combine in gradually reducing the resistance of the structure requires the development of simplified procedures for the evaluation of the combined damage accumulation, which embody both the physical and probabilistic aspects of design.

The present report attempts to develop the basis for an approach to the solution of this problem, for which at present no adequate experimental information exists, and one of its purposes is to provide the guidelines for the planning of tests and experiments, the results of which would be relevant for structural design.

This report has been reviewed and is approved.

W. J. Trapp

W. J. Trapp
Chief, Strength and Dynamics Branch
Metals and Ceramics Laboratory
Directorate of Materials and Processes

Table of Contents

Page

I. Introduction 1

II. Aspects of Design 5

III. Aspects of Damage Accumulation in Creep and Fatigue at Elevated Temperature 10

IV. Interaction Curves 16

V. Reliability and Risk of Failure 23

Bibliography 31

List of Figures

Figure		Page
1	Comparison of Creep Under Sustained and Under Periodically Repeated Stress Involving First-Stage Creep	32
2	Comparison of Creep Under Sustained and Under Periodically Repeated Stress Involving Second-Stage Creep	33
3	Elliptic Interaction Curves for Failure at Various Temperature	34
4	Interaction Surfaces for Failure	34
5	Interaction Surfaces for Limiting Creep-Deformation	35
6	Intersection of Interaction Surfaces for Failure and Limiting Creep Deformation (Design Ranges)	36
7	Interaction Surfaces for Failure at Various Probabilities of Survival Reliabilities R	37
8	Reliability Surface $R_\alpha(S,t)$ at Constant Angle $\alpha = \arctan (S_m^\alpha/S_a)$	38
9	Reliability Surface $R_S(\alpha,t)$ at Constant Stress Intensity $S = \sqrt{S_m^2 + S_a^2}$	39

I. Introduction

Among the many problems facing the designer of supersonic aircraft structures, space structures and missiles, the most serious appears to be that of attaining adequate safety and reliability under conditions of operation at "elevated temperatures." Such conditions arise only for combinations of structural material and temperature for which the repeated temperature and loading cycles either produce significant changes in the deformational (elastic and inelastic) response and in the mechanical strength of the material, or appreciably affect the stress distribution within a redundant structure. Thus, for instance, for the structural alloys used at present, "elevated temperature" conditions would arise in a supersonic transport above Mach 2 if constructed of light-weight alloys, or above Mach 3 if constructed of steel or titanium alloys.

In the design of power generators and jet engines, "elevated temperature" conditions are usually met by a combination of the use of alloys of high long-time temperature strength with extensive testing of full-scale structural parts under critical conditions so as to permit statistical evaluation of the test results as a basis for reliability analysis. Moreover, in such structures the creep deformation tolerances are rather high and the expected service lives of the critical parts relatively short, so that testing is possible within the operational conditions.

For large structures such as aircraft, space structures or missiles, statistically adequate test replication

---

Manuscript released by author November 1962 for publication as an ASD Technical Documentary Report is neither technically nor economically feasible, so that reliability analysis cannot be based on the evaluation of statistical data but must be developed from physically relevant probability concepts. This is a procedure similar to that suggested for the analysis of reliability and fatigue sensitivity of subsonic aircraft structures at normal atmospheric temperatures,[1] with the significant difference, however, that under elevated temperature conditions the damaging effects of fatigue and creep must be combined.

An additional difficulty arises from the fact that in the design for elevated temperature service over a life range of 10,000 to 100,000 hours the double aspect of design for failure and design for a maximum tolerable deformation requires that the tolerable creep rates per hour be extremely low while the times to fracture become extremely long. A survey of the considerable amount of existing creep data will, however, show that reliable observations within the structurally relevant range of low creep rates are almost nonexistent, while the recorded times to creep fracture are too short to be relevant in the design for the life range of supersonic aircraft structures which is too long to be used as actual testing time. The discouraging conclusion that can be reached from this survey is, therefore, that data for elevated temperature design of such structures for the relevant range of service lives are not available and could only be obtained from the available data by extrapolations, the reliability of which both in the direction of very low creep rates as well as of very long creep-rupture lives is rather dubious.[2] With most creep and creep-rupture tests performed within the 300 hr. range, extrapolation over two orders of magnitude in time would be necessary. This would be an inadmissibly wide range of extrapolation even for thermally and chemically stable metals, which the alloys developed for high temperature service are definitely not. The use of the various time-temperature-parameters suggested by different authors, and essentially based on the assumption that a single rate-equation covers the whole range of behavior can therefore hardly be expected to produce reliable data by extrapolation, unless the validity of this assumption has been established. This can, however, not be done directly without extending the testing time towards the actual service time. It might be attempted by introducing the assumption that the mechanical behavior is covered by two rate equations, one below and one beyond an instability time to temperature so that there is a time-temperature equivalence for the instability point, which would permit the replacement of long-time tests by high temperature tests.

It is well-known that the lack of reproducibility of creep observations within the (primary) micro-creep range makes prediction on the basis of a small number of tests completely unreliable: the scatter associated with (primary) micro-creep rates is wider than the scatter associated with (secondary) minimum steady-state creep rates and creep fracture data, so that designs for long lives in which a small total creep deformation (0.5-1%) represents a critical design criterion are subject to a high degree of uncertainty.

While actual data for the establishment of design procedures under conditions of elevated temperature are thus lacking, in spite of many years of creep research, it seems desirable to establish a general scheme for such a procedure in the light of which the current and future research work in creep and creep fracture for the purposes of structural design of supersonic aircraft and space structures should be re-evaluated and re-directed so as to provide the information that is relevant to design problems rather than to materials evaluation, for the purpose of which relatively short-time tests are usually considered expedient and sufficient. It is to be feared that without such a scheme the haphazard approach to the study of mechanical properties of metals at elevated temperatures will continue to remain unrelated to the needs of the designer. It is not adequately understood by metallurgists that for the structural designer creep and creep fracture are not metallurgical problems alone, but problems of reliability of a structure under highly variable conditions of stress, temperature and environment. Many of the materials involved are known to have a structure that tends to be unstable under critical combinations of time, temperature and environment, a phenomenon the occurrence of which cannot be predicted by extrapolation from the stable conditions in the usual short-time tests at constant stress and temperature. But even under stable conditions the scatter of the test results is to the designer as important as their trend. Since even small numbers of full-scale tests of large structures under elevated temperature conditions cannot be performed, the scatter of small specimen tests represents the only source of direct information on the basis of which procedures of reliability analysis can be devised.

It appears, therefore, that an urgent task of engineering research is the formulation of an integrated, schematic approach to the mechanical design of aircraft structures under elevated temperature service that would combine the safety-factor approach with the reliability approach and would also specify the test conditions from which the relevant quantitative information on material behavior could be obtained, and on the basis of which it may become possible to estimate the service life of a designed structure and to assess the effect on this service life of modifications of the design criteria as well as of changes in operating conditions.

II. Aspects of Design

The main aspects of design for elevated temperature service of aircraft and space structures are the interactions of sustained loads, cyclic loads and temperature in producing (a) a limiting, relatively small amount of permanent deformation within the service life of the structure; (b) damage accumulation in the form of micro- or macro-cracks that reduce the resistance of the structure to "ultimate" loads, as well as (c) the interaction of stress-relaxation, temperature and cyclic stress in modifying the structural action of redundant structures. Because of the highly statistical character of gust- and maneuver loads, it is reasonable to assume that, as under conditions of fatigue alone, elevated temperature creep-fatigue failure is more likely to occur under an excessive load of short duration and relatively low probability of occurrence after the structural resistance has been reduced by creep-fatigue damage, then under normal service conditions involving only loads of relatively low intensity occurring under such conditions. Hence the critical design conditions for elevated temperature service are: (a) ultimate load failure of the undamaged structure under a load intensity of very low probability of occurrence or of a "return period" that is a large multiple of the design life; (b) ultimate load failure of the structure damaged by fatigue and creep and therefore failing under an "ultimate load" of lower intensity but higher probability of occurrence or shorter "return period"; (c) limiting permanent deformation produced by creep- and fatigue loads under operating conditions specified in terms of load-spectra and temperature history.

It appears, therefore, that tests to creep-rupture under sustained stress with and without superimposed constant-amplitude cyclic stress are not quite relevant to the actual design problem, which is that of structural damage by crack-initiation and propagation under relatively low-intensity loads of long duration reducing the resistance to loads of high intensity and short duration. For this purpose tests under combinations of sustained and cyclic loads should be performed not to failure but to a specified level of reduced resistance measured either directly by a high strain rate test to failure or by a specified crack-length. Results of tests to failure under stationary conditions of sustained and cyclic loading can only provide information concerning the general trend of the phenomenon of damage, but the significant aspect for design is the fact that actual failure is most likely to be "premature," because of the possible coincidence of a partially damaged (cracked) structure with a load cycle which is just high enough to make this damage critical; the more severe the preceding damage, the lower the critical load and the higher therefore its probability of occurrence or the shorter its "return period." If the resistance of the structure is reduced to a level close to "limit" load, the return period of which is of the order of magnitude of the service life, the probability of failure during the service life becomes excessively high.

Similarly, the results of conventional creep tests are not quite relevant to the problem of creep design, since the very limited amount of operationally permissible total creep deformation makes the region of primary micro-creep of principal importance while the emphasis in conventional creep tests is on the steady-state (secondary) macro-creep. Not only is the primary micro-creep deformation of polycrystalline metals highly statistical in itself, but the accuracy of the measurement required in this range is usually far beyond that of conventional creep tests. The experimental difficulties arising in observations within this region are clearly demonstrated by the lack of reproducibility in stress-relaxation tests of the initial short-time range which depends primarily on the application of the initial strain which, being of the order of magnitude of the elastic strain, falls into the range of micro-creep. Since this lack of reproducibility cannot be overcome even by the most careful experimental technique, all stress relaxation tests of metals are unreliable within this, for the designer most important, range.

The experimental difficulties are magnified by the fact that, physically, primary creep in polycrystalline metals is the result of the combined action of and interaction between an irrecoverable component of strain-hardening associated with transcrystalline slip and a (delayed recoverable) anelastic component associated with grain-boundary motion; the former is nonlinear (with respect to stress), the latter linear. Hence creep tests without unloading periods, with the aid of which the anelastic component can be separated from the elastic and strain-hardening components, provide inadequate information for the designer who is unable to arrive at the analytical characterization of the material required for stress-analysis without such separation. Moreover, it appears that even the highest accuracy of direct observation of micro-creep and creep-recovery can not cover the short range relaxation range, and, to separate the linear anelastic from the non-linear components making up the total creep, the direct creep measurements should be supplemented by observation of the mechanical response of the material under low-frequency small-amplitude stationary oscillations, by which the relevant frequency dependent visco-elastic parameters (storage and loss moduli) can be obtained. Conversion of these data into creep- and relaxation functions representing the anelastic component of the creep-deformation is permissible, provided the time-scales are comparable.

In the interpretation of creep-deformation of a particular metal for the purposes of design two basically different conditions defined in terms of stress-level and temperature must be distinguished: (a) conditions that produce the limiting creep deformation essentially in the form of primary creep (Fig. 1); (b) condition that produce the limiting creep deformation essentially in the form of stationary (secondary) creep (Fig. 2).

In the first case a significant part of the limiting creep will be reached relatively early in the service life with little subsequent permanent deformation, while in the second case creep will accummulate rather uniformly over the service life.

In Figs. 1 and 2 the differences are schematically illustrated between creep under sustained stress and creep accumulation due to periodically applied stress of the same intensity under conditions of thermal stability and thermal instability (recovery). For both types of creep thermally unstable conditions of the periodically stressed material produce more rapid creep-accumulation than is to be expected from tests under thermally stable conditions, because of the effect of recovery on primary creep. Such increase in creep accumulation is, however, not necessarily accompanied by creep-damage accumulation, since the mechanism of primary creep is one associated with strengthening rather than weakening of the structure.

The methods of stress-analysis and the resulting stress-intensities and distributions will differ significantly for both conditions and so will the damage accumulation due to combined steady and cyclic loading: in the first case damage is predominantly due to cyclic loading while the effect of primary creep on damage is likely to be minor; in the second case damage accumulation is due to creep while fatigue-damage, when the cyclic stresses are high enough to produce it, proceeds independently until, at a rather advanced stage of propagation and coalescence of micro-cracks, the coexistence of both intercrystalline (creep) and transcrystalline (fatigue) micro-cracks is likely to facilitate the propagation of the macro-crack producing final failure through both crystal grains and grain-boundaries. With increasing temperature and service life creep-damage accumulation as the critical failure mechanism may grow so fast in relative importance in comparison to fatigue damage accumulation that beyond a certain combination of temperature and service life the weakening of the structural resistance due to creep becomes the principal design criterion while fatigue cracking is less critical as the working stresses of the structure are reduced to meet the limiting creep and creep-rupture criteria.

III. Aspects of Damage Accumulation in Creep and Fatigue at Elevated Temperature

It is currently believed that the main sources of room-temperature fatigue damage are the formation of persistent broad slip-bands, of pores, surface intrusions and extrusions formed by combined reversed primary slip and cross-slip, grain-boundary blocking of reversed micro-slip and, finally, exhaustion of ductility resulting from reversal of over-all large plastic strain. It is also believed that the sources of creep damage are sub-structure formation and grain-dissociation, as well as grain-boundary cracking du sliding and rotation along grain-boundaries incompatible with the continuity of the material.

The effect of elevated temperature on fatigue damage accumulation and fatigue life may be interpreted as the result of its modifying influence on the deformation processes preceding and accompanying the initiation of fatigue cracks at room temperature, as well as on the propagation of cracks that have attained macroscopic size. The deformation processes presumably involved in fatigue damage accumulation are reversed slip on primary slip planes, cross-slip, subgrain formation and hardening close to the tip of the advancing crack, and, finally, changes of deformational characteristics resulting from thermal instability and metallurgical changes in the microstructure. Possible modifications in the macrocrack propagation mechanism may arise from the increased time-sensitivity of the gross deformational response of the bulk metal at elevated temperature due to the effect of grain-boundary viscosity, and from the resulting (thermal) instability of the (compressive or tensile) residual and micro-residual stresses that either impede or facilitate the propagation of a crack, being responsible for the existence of the "Bauschinger effect" at room and low temperatures. Because of the instability of residual stresses at elevated temperatures all processes by which the fatigue life is increased as a result of mechanically or structurally induced compressive residual surface stresses (shot-preening, cold-rolling, carburizing, etc.) become largely ineffective.

The dependence of the deformation processes on the initial condition of the grain structure requires that distinction be made between fully annealed structures on the one hand and structures in various conditions of initial hardening on the other. The continuous modification of the grain structure by sufficiently elevated temperatures and by sustained mean loads requires, moreover, the careful consideration, at various temperatures, of the interaction of creep deformation and fatigue deformation and of the resulting micro-crack formation under the joint action of sustained and cyclic loads. Since the significance of the temperature effect necessarily increases with increasing mean stress and increasing time, and is therefore more pronounced with respect to creep (time-dependent) damage than to fatigue (cycle-dependent) damage, and more pronounced for long-time than for short-time behavior, this effect will vary with varying ratios of mean stress to amplitude of stress cycles, as well as with the absolute cyclic stress amplitude, being at its peak for long-time creep (zero cyclic stress). The ratio of the stress amplitude for very long fatigue life to the stress amplitude for very short life will therefore decrease with increasing temperature as well as with decreasing ratio of cyclic stress-amplitude to mean stress, being the lower the higher the temperature and the closer the conditions to those of creep-rupture. Hence at the same applied stress amplitude the fatigue life in number of cycles necessarily decreases with decreasing cycle frequency as well as with increasing temperature, producing a moderate increase of the slope of the S-N-diagram with temperature and with mean stress. Some isolated test results on notched high-temperature alloys (PH15 - 7 Mo.) in the temperature region around 800°F and $N = 10^3$ to 10^4 cycles suggest a reduction of the fatigue life by a factor of 3 if the frequency is reduced by 3 orders of magnitude (from 10^3 to 1 cycle per minute).

Considering that, in first approximation, the principal mechanism of fatigue damage accumulation operates in reversed slip and is therefore almost temperature- and time-insensitive, while the principal mechanism of creep-damage accumulation operates in grain-boundary dissociation and grain-boundary motion, both of which are highly temperature- and time-sensitive, it can be concluded that the same stress intensity will be the more damaging the more rapidly it can produce fatigue damage by reversed slip; this is the range of short fatigue lives at relatively high stress-amplitude and moderately elevated temperatures (about 1/4 to 1/3 T_m). In this range the ratio of [alternating stress/sustained stress] for which fracture is produced at the same time and temperature will always be less than one; with increasing life and temperature this ratio will increase, attaining values that are larger than one at high temperatures for very long lives and associated very low stress-intensities. Observed values of this ratio[3] vary between 0.6 - 0.8 for short lives at moderate temperatures and 1.0 - 2.0 for medium and long lives at high temperatures.

While the effect of temperature on reversed primary slip and damage accumulation in the fatigue striations is relatively weak, this cannot be assumed with respect to cross-slip, subgrain formation, and condensation of defects into holes and micro-fissures preceding the propagation of cracks through the subdivided grains. On the contrary, it may be assumed that elevated temperature facilitates cross-slip[4] so that the surface disturbances due to cross-slip are intensified. Similarly, the limiting stable subgrain-volume is a function of temperature, increasing considerably with increasing temperature; hence the subdivided structure as well as the resulting hardening and associated Bauschinger effect are the less stable the higher the temperature. While subgrain-formation is probably reduced or retarded as a result of elevated temperature grain-boundary migration and recovery, crack propagation through the subdivided structure may be facilitated by the higher subgrain-boundary mobility, provided precipitate particles do not hinder such motion. Since cross-slip is also facilitated, the associated surface disturbances leading to transcrystalline extrusions, intrusions and fissures become more extensive. Thus it can be assumed that not only the principal damage accumulation mechanism is gradually transferred into the grain boundaries with increasing temperature, so that even under relatively low mean stress the fatigue damage mechanism approaches more and more that of creep damage with resulting stronger interaction between both, but also the intensification of cross-slip accelerates the over-all fatigue damage by intensifying transcrystalline surface damage.

It can therefore be expected that with increasing temperature the fatigue life both in number of cycles and in time at a certain combination of cyclic and sustained stress decreases monotonically unless a temperature range exists in which extensive slip-blocking is produced by metallurgical changes in the structure (embrittlement). Within this range the fatigue life will, in general, increase; this fact has been observed in low-carbon steels within the "blue-brittleness" range of 250° to 350° C[5] in which slip is impeded by the "dislocation pinning" effect of carbon or nitrogen.

The transition temperature range from predominantly transcrystalline fracture to predominantly intergranular fracture varies with stress-amplitude, mean stress, frequency of stress cycling and environment. It decreases with decreasing ratio of stress-amplitude to mean stress, with decreasing frequency and with increasing corrosive effect of the environment.

It should be realized that creep-damage accumulation by grain-boundary motion is characteristic of second-stage (quasi-viscous) and third-stage creep only. Where the deformation under mean stress involves mainly primary creep, as is the case, for instance, under periodically repeated sustained stress and thermally instable conditions of the structure (rapid recovery and thermal softening), very little damage accumulation in grain-boundaries is involved. The principal fatigue damage arises from reversed slip under cyclic stress amplitudes, with resulting rather limited effect of temperature on the fatigue mechanism, while the unidirectional slip associated with primary creep produces little, if any, damage.

Any effect that weakens the grain-boundaries, such as elevated temperature, migration of impurities or chemical attack not only reduces the time to creep-rupture at a specified stress level or reduces the sustained stress required to produce such failure at a specified time, but also facilitates fatigue crack propagation through subdivided sub-grain material in the vicinity of the crack; the same effect is produced by increasing the relative volume of boundary material. Moreover, weakening of the grain-boundaries indirectly facilitates the initiation and propagation of transcrystalline fissures by reducing the restraints on slip propagation imposed, at low temperatures, by grain boundaries. Such indirect action, however, seems to be less significant in its effect on fatigue strength or fatigue life than the gradual transfer of the operating damage mechanism from the crystalline to the intercrystalline regions. Since such transfer is the more pronounced, the higher the temperature and the longer the life or the lower the stress-amplitude, it appears that elevated temperature-fatigue will be the more time- and frequency-sensitive the higher the temperature and the lower the stress-amplitude. Thus for high temperatures and long lives (in terms of time) both fatigue and creep-rupture may show similar time- (frequency) and temperature-sensitivity, while at lower temperatures and shorter lives fatigue is much less time- (frequency) sensitive than creep-rupture. Because of the similarity of the fatigue damage and creep-damage mechanisms for long lives and high temperatures any structural modification producing high creep resistance (low creep rate) will also produce improved fatigue performance.

IV. Interaction Curves

Because of the practical absence of damage accumulation under very small sustained stresses the effect of creep damage on fatigue damage can be neglected for small ratios of sustained to cyclic stress; on the other hand, the effect of low-amplitude cyclic stresses on damage under sustained stress must be expected to vary with amplitude and frequency. High frequency cyclic stressing, by its interaction with micro-structural processes may significantly affect the creep-rate and associated damage accumulation either by accelerating the rate of recovery and of thermal softening in a work-hardened metal or by accelerating metallurgical transformations such as precipitation, leading to hardening or other forms of atomic migration. Hence the result of a superimposed high frequency low-amplitude oscillation may be either a significant decrease or an increase of the creep-strength, depending on the microstructures. Many creep resistant alloys such as cobalt and nickel alloys achieve this resistance by some form of atomic migration; in that case acceleration of this migration by superimposed cyclic stressing may considerably increase this resistance. On the other hand, the imposition of a cyclic stress of low or moderate frequency that does not interact with micro-structural processes will not significantly affect the creep-damage unless the stress amplitude is high enough to change the character of the damage mechanism from purely intergranular to a mixture of intergranular and transgranular. By relieving grain-boundary stress-concentrations and reducing the associated cracking, a cyclic stress of moderate amplitude may even increase the creep-rupture strength, a phenomenon that has actually been observed.[6] On the other hand, by substructure formation it may accelerate the grain-dissociation under sustained load and thus increase the creep-rate and decrease the life.[7] In the absence of low-amplitude interaction, "interaction curves" representing the combined creep-fatigue damage under a sustained load S_m and superimposed load-cycles of amplitude $\pm S_a$ at a certain combination of time and temperature should, in the rectangular $S_m - S_a$ coordinate system, be tangent to the rectangle $S_a = S_{ao}$ and $S_m = S_{mo}$, where S_{mo} and S_{ao} denote, respectively, the sustained stress and the stress amplitude applied at frequency ω, producing failure or a specified amount of damage (crack-length or percentage loss of "ultimate" strength) at a certain time t or at a number of cycles $N = \omega t$ and at temperature T (Fig. 3). In general however, the condition of the existence of a vertical tangent at S_{mo} is not physically justified; the interaction curve may intersect the S_m - axis at an acute (thermal softening) or obtuse (precipitation hardening) angle.

Since the existing experimental evidence suggests[3] that for relatively short times and moderately elevated temperatures the ratio $S_{mo}/S_{ao} > 1.0$ while it decreases to $S_{mo}/S_{ao} < 1.0$ as the temperature and time increase, a damage or fracture surface at temperature T in a rectangular $S_a - S_m - t$ coordinate system can be constructed by assuming elliptic "interaction curves" and connecting the elliptic interaction curves for different times $t = N/\omega$ by a continuous surface (Fig. 4). The intersection of this surface with the $S_m - t$ plane is the creep-rupture curve (for failure) or the locus of points of specified constant damage for various combinations of stress and time (at constant temperature); the intersection of the surface with the $S_a(t = N/\omega)$ plane is the S_a - N-diagram for failure or specified damage converted to the equivalent time scale at a specified frequency.

It is important to note that the elliptic or quasi-elliptic shape of the interaction curves can be justified only under elevated temperature conditions under which creep damage actually accumulates under the sustained stress S_m . These interaction curves should not be confused with the pure fatigue-interaction curves at normal temperatures (modified Goodman diagrams) as there are no different damage mechanisms involved. Indeed these interaction diagrams are frequently linear or even convex towards the S_m axis. Moreover, in this case, the time-cycle conversion $t = N/\omega$ is purely fictitious since, in the absence of creep, time itself does not contribute to damage accumulation. Therefore constant creep-fatigue damage surfaces cannot be extended to conditions of zero or non-damaging (first stage) creep.

If it were possible to find a combined time-temperature-frequency parameter $\theta(t,T,\omega)$ that would express the time-temperature-frequency equivalence jointly for creep and fatigue, all constant-damage surfaces for various temperatures T could be combined into a single "master" surface in the S_a - S_m - coordinate system. For instance, the normal high temperature creep-rupture correlation parameters of the form $\theta(t,T) = t^r e^{s(T)}$ could be expanded by introducing, for instance, $\theta(t,T,\omega) = t^r e^{s(T)}(\omega/\omega_o)^n$, where ω_o denotes a (very low) reference frequency for which $\theta(t,T,\omega) \doteq \theta(t,T)$. Adequate experiments do not exist, however, from which the frequency effect in $\theta(t,T,\omega)$ could be evaluated so that, for the time being, the surfaces for various temperatures cannot be unified through a combined parameter.

The creep strain attained at a certain time under a combination of sustained stress S_m and cyclic stress-amplitude S_a is, if at all, only slightly and indirectly affected by the cyclic stress. Hence the "interaction surfaces" for constant creep strain are either parallel to the S_a - axis or slightly inclined to it if the stress-cycles are associated with a slight permanent strain, or as is usually the case, because of the plastic deformation accompanying the first few stress-cycles (Fig. 5). The intersection of the constant creep or strain surfaces at the same temperature with the constant damage surfaces delimits the validity of the two design criteria (Fig. 6). It appears that for short or moderate times and small or moderate ratios of the "fatigue factor" $\tan \alpha = S_a/S_m$ the creep-deformation criterion will be relevant, while for all other conditions damage accumulation is more significant. Obviously, the limit varies with time and temperature.

On the basis of the constant damage surface $f(S_m, S_a, t) = 0$ damage accumulation at various combinations (S_m, S_a) could be estimated, in first approximation, by finding the time t_d for a particular combination at which the point of intersection of the radius vector of slope $\tan \alpha = S_a/S_m$ in the plane $t = t_d$ parallel to the S_m - S_a plane, with the damage surface has the required combination (S_m, S_a) as coordinates, and by assuming a linear rule of damage-accumulation with respect to this time. Under this assumption the expression

$$\sum_{k=1}^{k=n} (t_k/t_{dk}) = \sum_{1}^{n} \frac{p_k t_d}{t_{dk}} = 1 \qquad (4.1)$$

where $t_{dk}(\alpha)$ are the times to constant damage or to failure at the various combinations $(S_m, S_a)_k$, and p_k the percentage of total time t_d during which they are applied. It is likely that interactions between the various combinations exist so that the times t_{dk} should be reduced with the aid of interaction factors w_k to (t_{dk}/w_k) if the combination $(S_m, S_a)_k$ forms part of a load spectrum instead of being continued to failure, but no experimental information on this point is in existence, so that no design rules for variable conditions can be given.

In the absence of surfaces of failure or of constant damage the times t_{dk} are unknown and must be estimated from the time t_c to creep fracture under S_m and the number of cycles N or time N/ω to fatigue fracture under $\pm S_a$. On the crude basis of assumed independence between the two damage processes a tentative rule of addition of linear damage rates would produce the relation

$$\frac{1}{N/\omega} + \frac{1}{t_c} = \frac{1}{t_d} \tag{4.2}$$

which could be improved and adjusted to experimental results by the introduction of a factor $w_c(\alpha)$ that would express the relative weight of creep damage in relation to fatigue damage which is necessarily a function of $\alpha = \arctan (S_a/S_m)$; hence,

$$\frac{1}{t_d} = \frac{1}{N/\omega} + \frac{w_c(\alpha)}{t_c} . \tag{4.3}$$

It can be shown that the above damage rules are compatible with exponential distributions of creep and fatigue lives

t_c and N.[8] For different distributions as, for instance, asymptotic distributions of extreme values linear accumulation and addition is no longer justified but should be replaced by a rule of the type[9]

$$\frac{1}{t_d} = \sum^{i}\left(\frac{1}{N_i/\omega_i}\right)^{\beta_{fi}} + \sum^{k}\left(\frac{w_{ck}(\alpha)}{t_{ck}}\right)^{\beta_{ck}} \tag{4.4}$$

where β_{fi} and β_{ck} are inversely proportional to the standard deviations of log t in creep rupture tests and log N in fatigue tests at various stress-amplitudes S_{ai} and levels S_{mk}.

The interaction surface associated with the assumption of linear addition of the constant damage rates ω/N and $1/t$ leading to Eq. (4.2) can be obtained by assuming the relations between S_a and N and S_m and t in the form[10]

$$S_a = S_{ao}N^{-\alpha} \quad \text{and} \quad S_m = S_{mo}t^{-\beta} \tag{4.5}$$

where $\alpha < 1$, $\beta < 1$ and S_{ao} and S_{mo} are the values of S_a and S_m for $N = 1$ and $t = 1$ respectively, so that

$$N = (S_{ao}/S_a)^{1/\alpha} \quad \text{and} \quad t = (S_{mo}/S_m)^{1/\beta} .$$

Hence

$$\frac{\omega}{N} + \frac{1}{t} = \omega\left(\frac{S_a}{S_{ao}}\right)^{1/\alpha} + \left(\frac{S_m}{S_{mo}}\right)^{1/\beta} = \frac{1}{t_d} = \text{const.} \tag{4.6}$$

which for constant times t_d defines a family of interaction curves of pseudo-elliptic type:

$$t_d\omega\left(\frac{S_a}{S_{ao}}\right)^{1/\alpha} + t_d\left(\frac{S_m}{S_{mo}}\right)^{1/\beta} = 1 \qquad (4.7)$$

With increasing temperature α increases slowly, β fast; hence $1/\alpha$ remains unchanged while $1/\beta$ decreases, increasing the effect of creep.

Since at present neither the interaction surfaces nor the parameters of Eq. (4.4) are known from experiments, Eq. (4.2) or if $\omega_{ck}(\alpha)$ can be estimated, Eq. (4.3) or Eq. (4.7) provide the only quantitative rules for an estimate of the effect of damage accumulation in fatigue and creep on the fatigue life at elevated temperature. Since the form of Eq. (4.7) does not contradict the existing experimental results it might be concluded that Eqs. (4.2) or (4.3) provide an acceptable tentative rule of damage interaction.

V. Reliability and Risk of Failure

From the designer's point of view the constant-damage and failure surfaces $f(S_m, S_a, t) = 0$ as well as the constant creep surfaces represent the trend or "central tendency" (mean, mode or median) of the test results which should be represented by families of such surfaces (Fig. 7), each associated with a different probability of failure P or of survival (reliability) $R = 1 - P$. What the designer needs is the surface associated with a specified, very small value of P (or a large value of R) rather than the surface of central tendency at which $P \sim R$. From the family of failure-surfaces $f_R(S_m, S_a, t) = 0$ a four-dimensional reliability function $R(\alpha, s, t)$, where $\alpha = \arctan (S_a/S_m)$ and $S = \sqrt{S_m^2 + S_a^2}$ could theoretically be constructed point by point by determining for values $0.01 < R < 0.99$ the values t associated with the points of intersection with all those surfaces of a parallel to the t-axis through any combination (α, S). This reliability function specifies the probability of survival as a function of time $t = N/\omega$ of any applied combination (α, S) ; for $\alpha = 0$ it degenerates into the three-dimensional reliability surface $R_C(S,t)$ for creep-rupture under $S = S_m$, for $\alpha = \pi/2$ into the reliability surface for fatigue $R_F(S,N)$ under $S = S_a$ for a specified frequency of oscillation. For intermediate constant values $0 < \alpha < \pi/2$, as well as for specified values $S = \text{const.}$ the reliability function can be represented in the form of reliability surfaces $R_\alpha(S,t)$ for constant α and $R_s(\alpha,t)$ for constant values of $0 < S < S_{max}$, constructed through the family of reliability functions $R_\alpha(t)$ obtained by varying S at constant α, and $R_s(t)$ by varying α at constant S (Figs. 8 and 9).

If it is assumed that, in first approximation, α and S are independent statistical variables with a bi-variate frequency density $p(\alpha,S)$, the reliability-function $R^*(t)$ under random application of combinations (α,S) is obtained by performing the integration

$$R^*(t) = \int_0^{S_{max}} \int_0^{\pi/2} p(\alpha,S)R(\alpha,S,t)dS \cdot d\alpha \qquad (5.1)$$

provided analytic expressions for $p(\alpha,S)$ and $R(\alpha,S,t)$ can be specified.

It will be more expedient, however, to specify the independent marginal frequency densities $p_1(S)$ and $p_2(\alpha)$ rather than the bi-variate density $p(a,S)$, as well as the reliability surfaces $R_\alpha(S,t)$ and $R_s(\alpha,t)$ rather than the reliability function $R(\alpha,S,t)$. In this case a rough approximation of $R^*(t)$ might be obtained by

$$R^*(t) = \int_0^{S_{max}} \int^{\pi/2} p_1(S)R_\alpha(S,t)p_2(\alpha)R_s(\alpha,t)dSd\alpha. \qquad (5.2)$$

Since the bi-variate distribution $p(\alpha,S)$ is not uniquely determined by the specification of the marginal distributions[11] the integral in Eq. (5.1) cannot be determined unless $p(\alpha,S)$ can be derived from observations, which is much more difficult than the derivation of $p_1(S)$ and $p_2(\alpha)$ separately. Under these circumstances Eq. (5.2) seems to be the closest approximation to Eq. (5.1) that is obtainable.

The reliability function $R^*(t)$ for the distributions $p_1(S)$ and $p_2(\alpha)$ is based on the tacit assumptions of independence of S and α and linear damage accumulation under variable stress intensity S. It represents essentially the reliability function for the mean stress-intensity S and the mean fatigue factor α. As such it is only a rough approximation of reality, but without it not even the order of magnitude of the service life can be predicted.

Such prediction can now be based on the procedures proposed in WADD TR 61-53 and the concept of the addition of risk-functions developed there. The risk function $r^*(t)$ associated with the reliability function $R^*(t)$ is defined by the relation[12]

$$r^*(t) = -\frac{d}{dt} \log R^*(t) \qquad (5.3)$$

and can thus be obtained if $R^*(t)$ can be specified by an analytic function. The risk function for ultimate failure r_u and its relation to the safety factor ν for such failure has been extensively discussed in the report referred to, in which also the effect on ν of a propagating fatigue crack and its related effect on r_u has been treated. While extensive information is available on the propagation of fatigue cracks at room or low temperatures, almost no information exists on crack propagation at elevated temperatures under the joint action of creep and fatigue on which a quantitative relation could be based that would express the reduction of resistance with time of a cross-section stressed at elevated temperature by the combined action of sustained and cyclic stresses. Thus no more can be done here than to point to this lack of vital information from which the risk function $r_u'[\nu(t)]$ and thus $r_u'(t)$, expressing the deterioration with time of the ultimate strength due to combined creep and fatigue cracking, would have to be obtained in order to establish the total risk of failure considering all conditions:

$$r(t) = r_u + r_u'(t) + r^*(t) \qquad (5.4)$$

where r_u is the (constant) risk of ultimate failure of the undamaged structure

$$r_u = kP_f \qquad (5.5)$$

P_f is obtained from the distribution function of the safety factor $P(\nu) = \int_0^{\nu} p(\nu)d\nu$ at the abscissa $\nu = 1$, while $k \ll 1$ is the ratio of the expected number of intensities within the "ultimate" load intensity spectrum (above limit load) to the total number of applied loads.

A measure of elevated temperature fatigue-creep sensitivity, similar to the measure of fatigue sensitivity proposed in WADD TR 61-53 can now be defined as the sum of the ratios

$$f(t) = f'(t) + f''(t) = \frac{r^*(t)}{r_u} + \frac{r_u'(t)}{r_u} \qquad (5.6)$$

where the first ratio can, in general, be assumed to be small in relation to the second. Values $f(t) \gg 1$ would designate structures of high sensitivity for which special design and testing procedures have to be developed, while structures with $f(t) < 1$ may be designed by standard procedures for ultimate load only.

For known shapes of the reliability and risk functions diagrams for the prediction of service life t or reliability at a specified life t similar to those computed in WADD TR 61-53 (Figs. 10a-d) could be constructed or the diagrams directly used replacing the variable N by t, provided the reliability functions are of Extreme Value type, an assumption for which similar physical-probabilistic arguments can be put forward as in the case of fatigue proper.[13]

Considering two extremal risk functions, one for fatigue and one for creep-rupture

$$r'(N) = \frac{1}{V_s}\left(\frac{N}{V_s}\right)^{\gamma-1} = \frac{\gamma}{V_s}\left(\frac{\omega t}{V_s}\right)^{\gamma-1} , \quad \gamma > 1 \tag{5.7}$$

and

$$r''(t) = \frac{\delta}{t_s}\left(\frac{t}{t_s}\right)^{\delta-1} , \quad \delta > 1 \tag{5.8}$$

where V_s and t_s are, respectively, the characteristic fatigue life and the characteristic creep-life, and γ and δ are the scale parameters of the associated reliability functions

$$R'(N) = \exp[(-N/V_s)^{\gamma}] = \exp[(-\omega t/V_s)^{\gamma}] \tag{5.9}$$

and

$$R''(t) = \exp[(-t/t_s)^{\delta}] \tag{5.10}$$

which are inversely proportional to the standard deviations $\sigma(\log N)$. and $\sigma(\log t)$, the combined reliability function

$$R^*(t) = R'(N)\,R''(t) = \exp[-(\omega t/V_s)^\gamma]\exp[-(t/t_s)^\delta] \quad (5.11)$$

The effect of the interaction can be illustrated by comparing the modes or another suitable measure of central tendency of the probability densities $p'(N)$ and $p''(t)$ with the respective measure of the combined function $p^*(t)$. Using the medians $\check{t}'$, $\check{t}''$ and $\check{t}^*$ rather than the modes for the sake of expediency the following expressions are obtained for these values from

$$\exp\left[-\left(\frac{\omega t}{V_s}\right)^\gamma\right] = \frac{1}{2} : \quad \check{t}' = \frac{V_s}{\omega}\,(\ln 2)^{1/\gamma} \qquad (5.12)$$

from

$$\exp\left[-\left(\frac{t}{t_s}\right)\right] = \frac{1}{2} : \quad \check{t}'' = t_s\,(\ln 2)^{1/\delta} \qquad (5.13)$$

and from

$$\exp\left[-\left(\frac{\omega t}{V_s}\right)^\gamma\right]\exp\left[-\left(\frac{t}{t_s}\right)^\delta\right] = \frac{1}{2} \qquad (5.14)$$

the superposition rule

$$\left(\frac{\check{t}^*}{\check{t}'}\right)^\gamma + \left(\frac{\check{t}^*}{\check{t}''}\right)^\delta = 1 \qquad (5.15)$$

from which $\check{t}^*$ can be evaluated. Introducing the ratio $\check{t}'/\check{t}'' = n$ Eq. (5.15) can be written in the form

$$\frac{1}{n^\delta}\left(\frac{\check{t}^*}{\check{t}''}\right)^\gamma + \left(\frac{\check{t}^*}{\check{t}''}\right)^\delta = 1 \qquad (5.16)$$

which indicates that for $n \gg 1$: $\check{t}^* \rightarrow \check{t}''$, while for $n \ll 1$: $\check{t}^* \rightarrow \check{t}'$. If $\check{t}'$ and $\check{t}''$ are of the same order of magnitude $(n \sim 1)$ an estimate of t^* is obtained from

$$\left(\frac{\check{t}^*}{\check{t}''}\right) = \left[1 + \left(\frac{\check{t}^*}{\check{t}''}\right)^{\gamma-\delta}\right]^{-1/\delta} < 1 \qquad (5.17)$$

Hence $\check{t}^* < \check{t}''$ or $\check{t}'$; however, the reduction is rather small, being a maximum for $\gamma = \delta$ for which $\check{t}^* = \check{t}''/\sqrt[\delta]{2}$. With the usual range of values $3 < \delta < 5$: $0.8\check{t}'' < \check{t}^* < 0.87\check{t}''$ a result that suggests a relatively weak interaction if the independence of the failure mechanisms implied in Eq. (5.11) is a valid assumption. With $\gamma = \delta = 1$ Eq. (5.15) degenerates into the assumption of linear interaction, which is therefore compatible only with pure chance failures with constant risk functions $r'(N) = V_s^{-1}$ and $r''(t) = t_s^{-1}$ and therefore incompatible with the physical character of the fatigue and creep-rupture processes. This conclusion obviously affects the validity of Eqs. (4.6) and (4.7), but only in so far as the values of the exponents are concerned.

The general conclusion to be reached from Eq. (5.16) is that whenever the creep life under S_m and the fatigue life under S_a are significantly different, the critical life is roughly equal to the shorter of the two; when they are of the same order of magnitude there is interaction which, however, reduces the critical life only moderately. Since this reduction is well within the scatter range it appears as if existing creep data and existing alternating stress fatigue data may provide sufficient information for design under combined action of creep and fatigue. While the effect of mean stress in fatigue can obviously not be disregarded, it would be of considerable interest to check, by adequate experiments, the implication of Eq. (5.16) that whenever the mean stress S_m is significant enough to effect the fatigue life $\check{t}'$, under $\pm S_a$, its direct effect in producing creep-damage is such that $\check{t}'' < \check{t}'$, with the result that creep design for S_m becomes critical.

Bibliography

1. Freudenthal, A. M., WADD Technical Report 61-53, July 1961.

2. Shimmin, K. D., WADD Technical Report 60-523, October 1960.

3. Allen, N. P. and Forrest, P. G., Session 4, Paper 1, Int., "Conference Fatigue of Metals," London, 1956.

4. Friedel, J., "Dislocations and Mechanical Properties of Crystals," J. Wiley and Sons, New York, p. 330-332, 1957.

5. *Loc*. *cit*. (3), p. 4.

6. Lazan, B. J., Chapt. 9 in "High Temperature Effects in Aircraft Structures," Agardograph No. 28, 1958. (N. J. Hoff, Ed.).

7. Kennedy, A. J., Rheologica Acta, Vol. 2, pp. 22-26, 1962.

8. Freudenthal, A. M., "IUTAM Colloquium on Fatigue," Stockholm, Springer, Berlin, p. 57, 1956.

9. *Loc*. *cit*. (8), p. 59.

10. *Loc*. *cit*. (8), p. 60.

11. Gumbel, E. J., J. Am. Statist. Assoc., Vol. 56, p. 335, 1961.

12. *Loc*. *cit*. (1), p. 20.

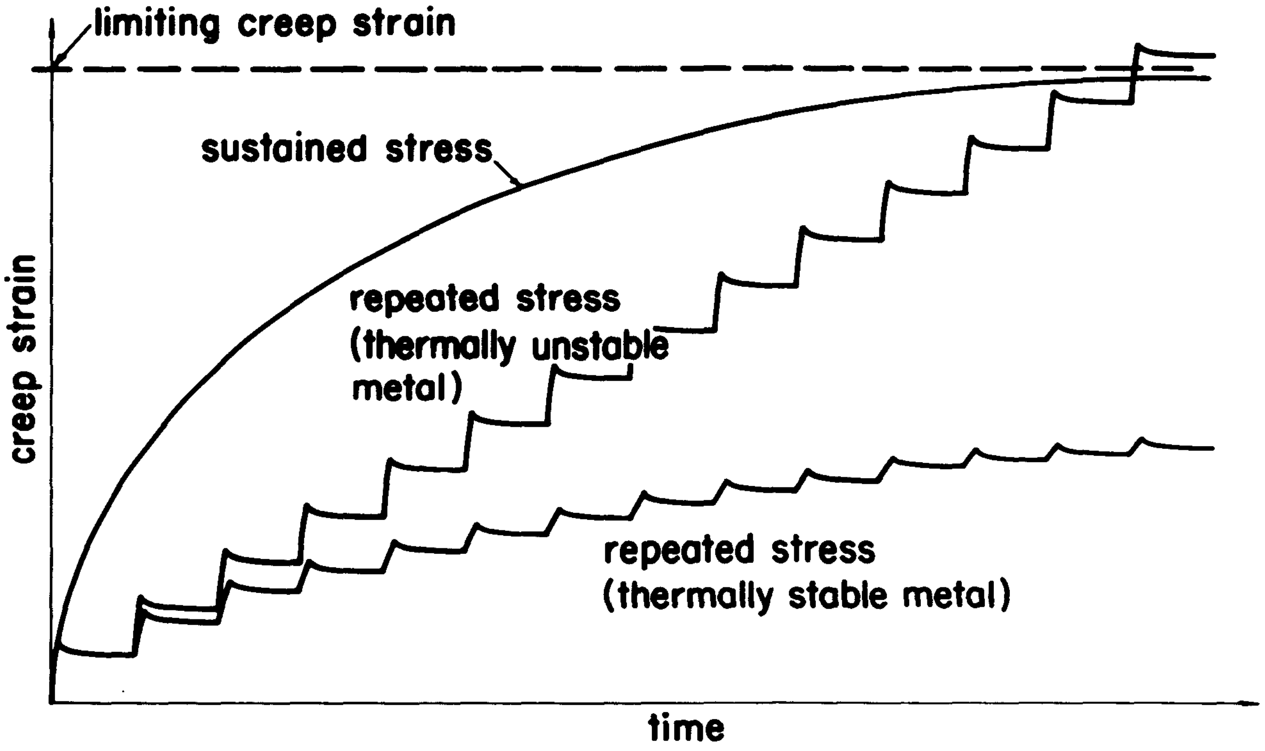

Fig. 1 Comparison of Creep Under Sustained and Under Periodically Repeated Stress Involving First-Stage Creep

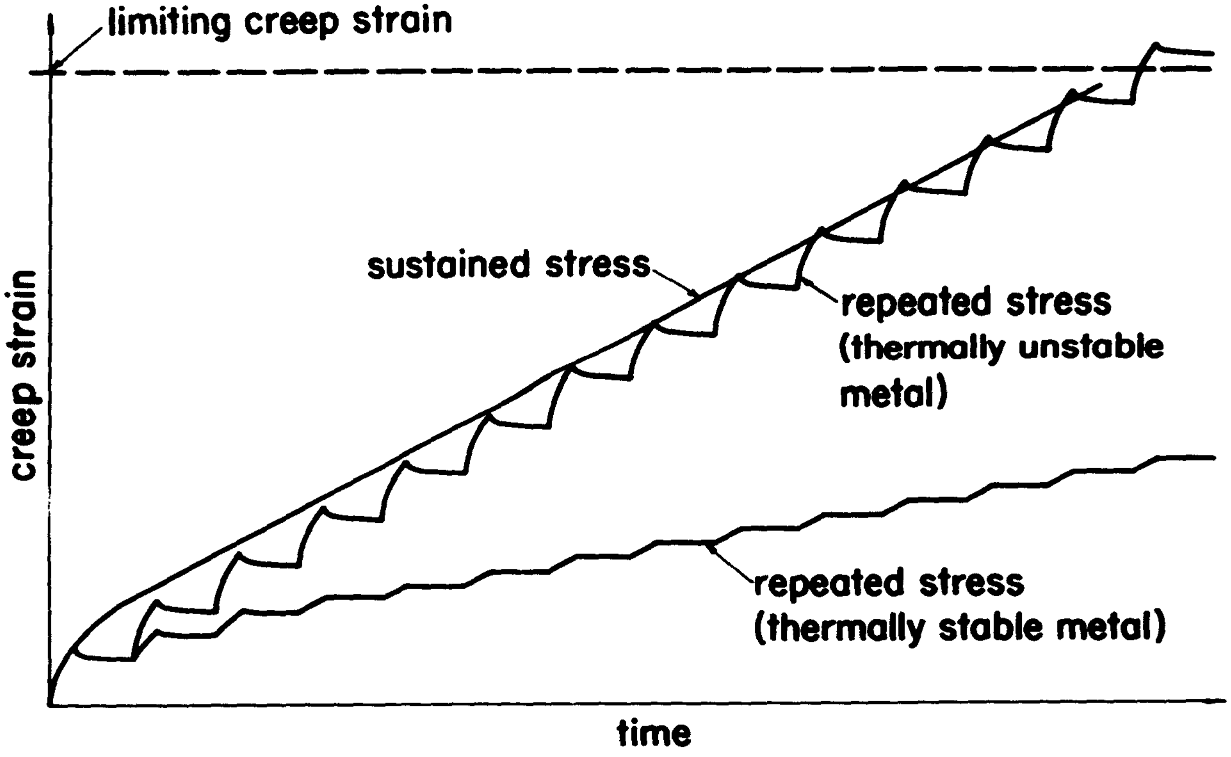

Fig. 2 Comparison of Creep Under Sustained and Under Periodically Repeated Stress Involving Second-Stage Creep

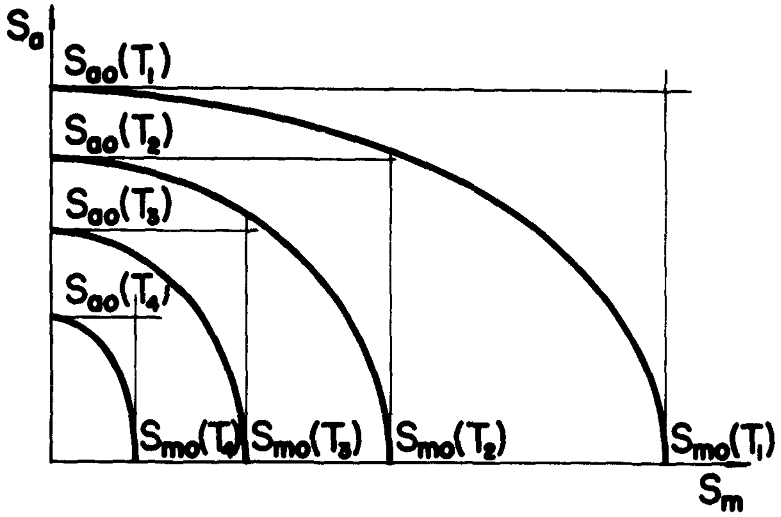

Fig. 3 Elliptic Interaction Curves for Failure at Various Temperature

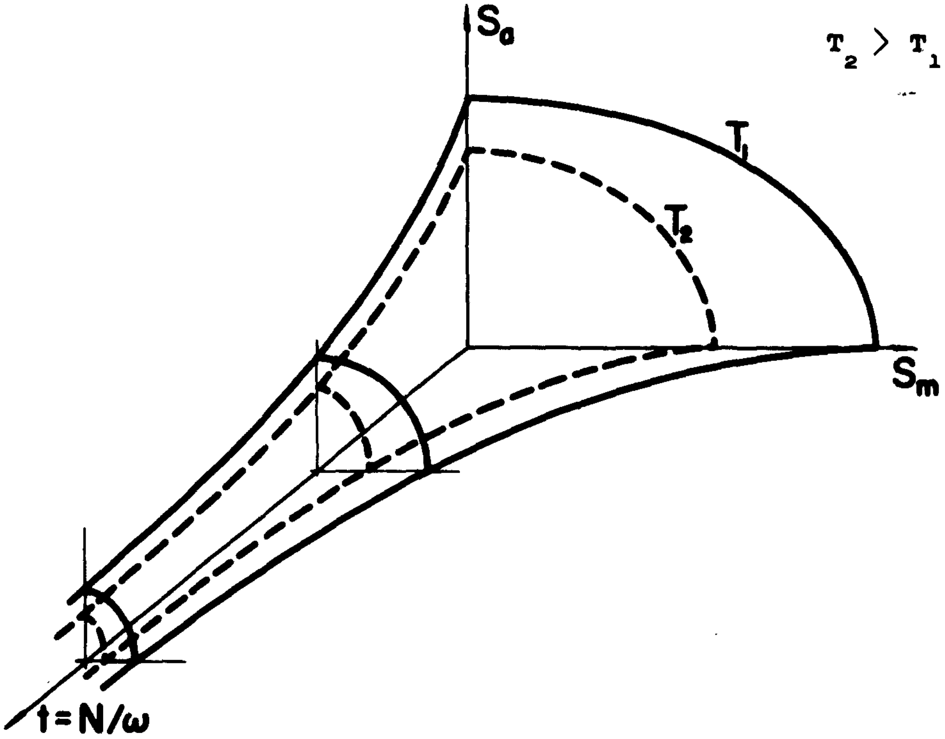
S_a
$T_2 > T_1$
T_1
T_2
S_m
$t = N/\omega$

Fig. 4 Interaction Surfaces for Failure

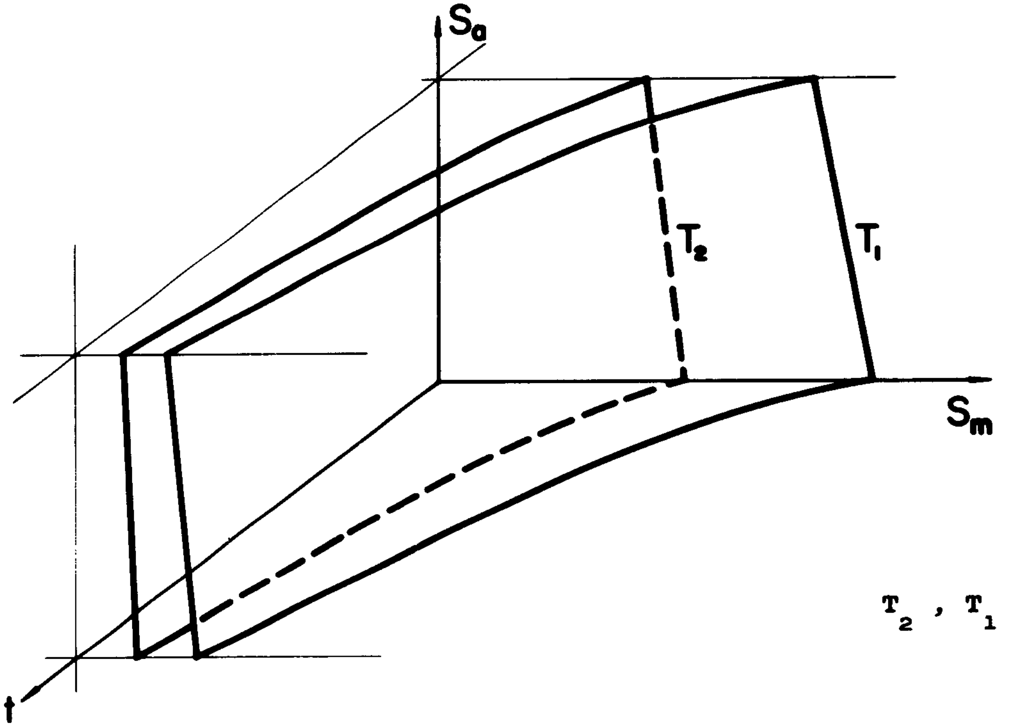
S_a
T_2
T_1
S_m
T_2 , T_1
t

Fig. 5 Interaction Surfaces for Limiting Creep-Deformation

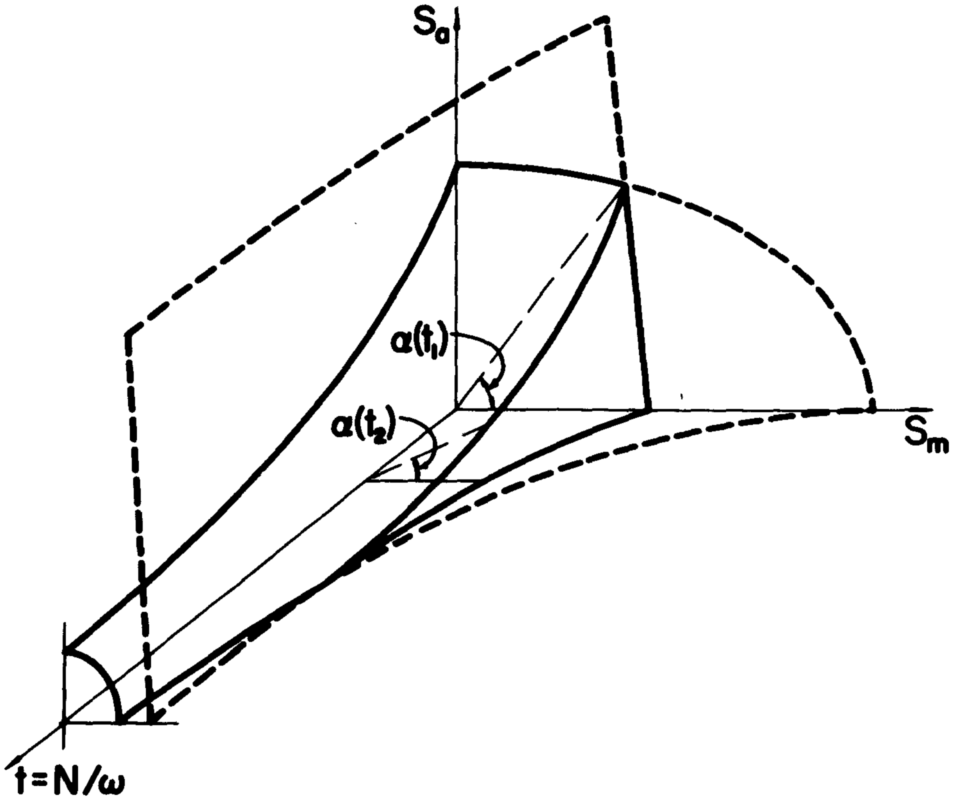
S_a
S_m
$\alpha(t_1)$
$\alpha(t_2)$
$t=N/\omega$

Fig. 6 Intersection of Interaction Surfaces for Failure and Limiting Creep Deformation (Design Ranges)

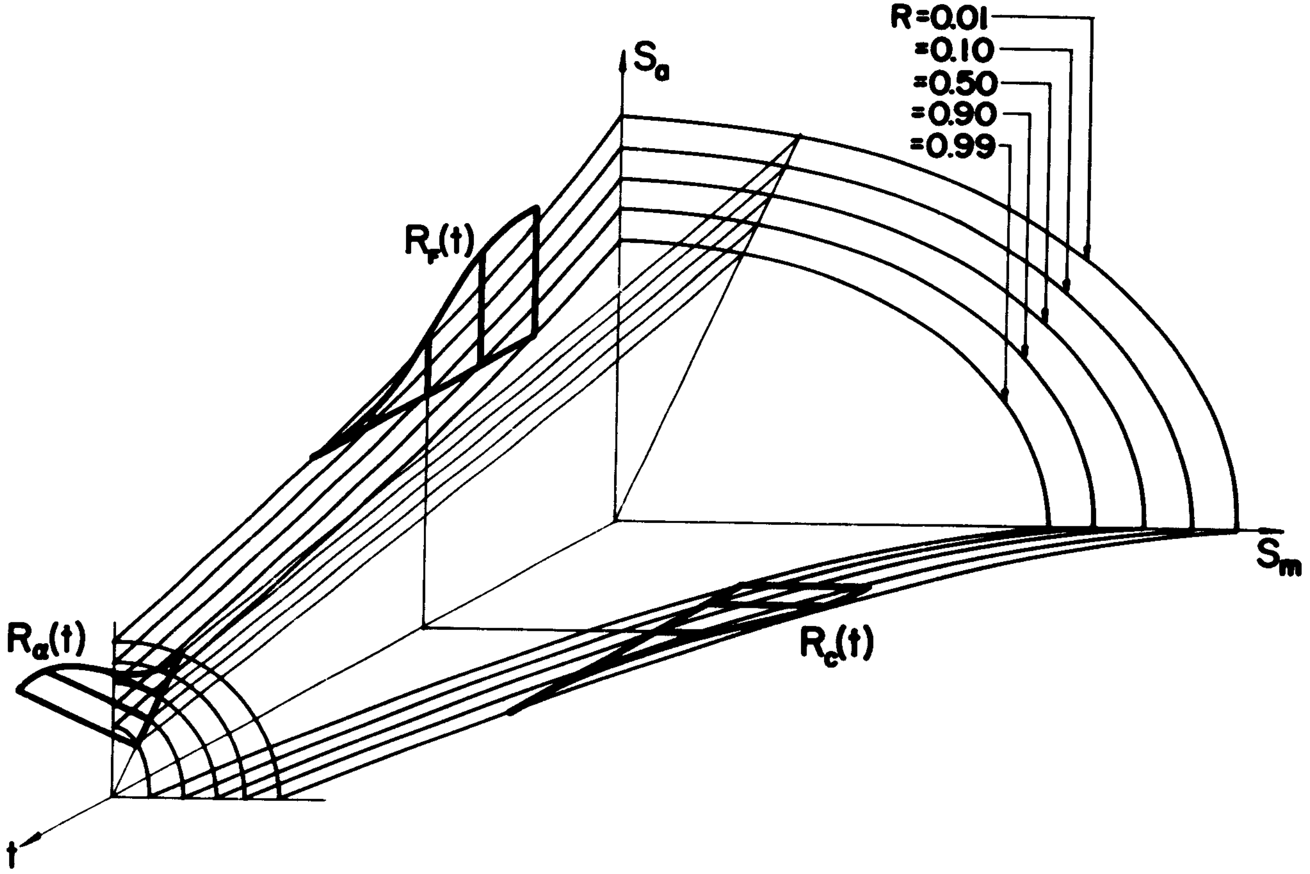
R=0.01
=0.10
=0.50
=0.90
=0.99
S_a
S_m
$R_r(t)$
$R_c(t)$
$R_a(t)$
t

Fig. 7 Interaction Surfaces for Failure at Various Probabilities of Survival Reliabilities R

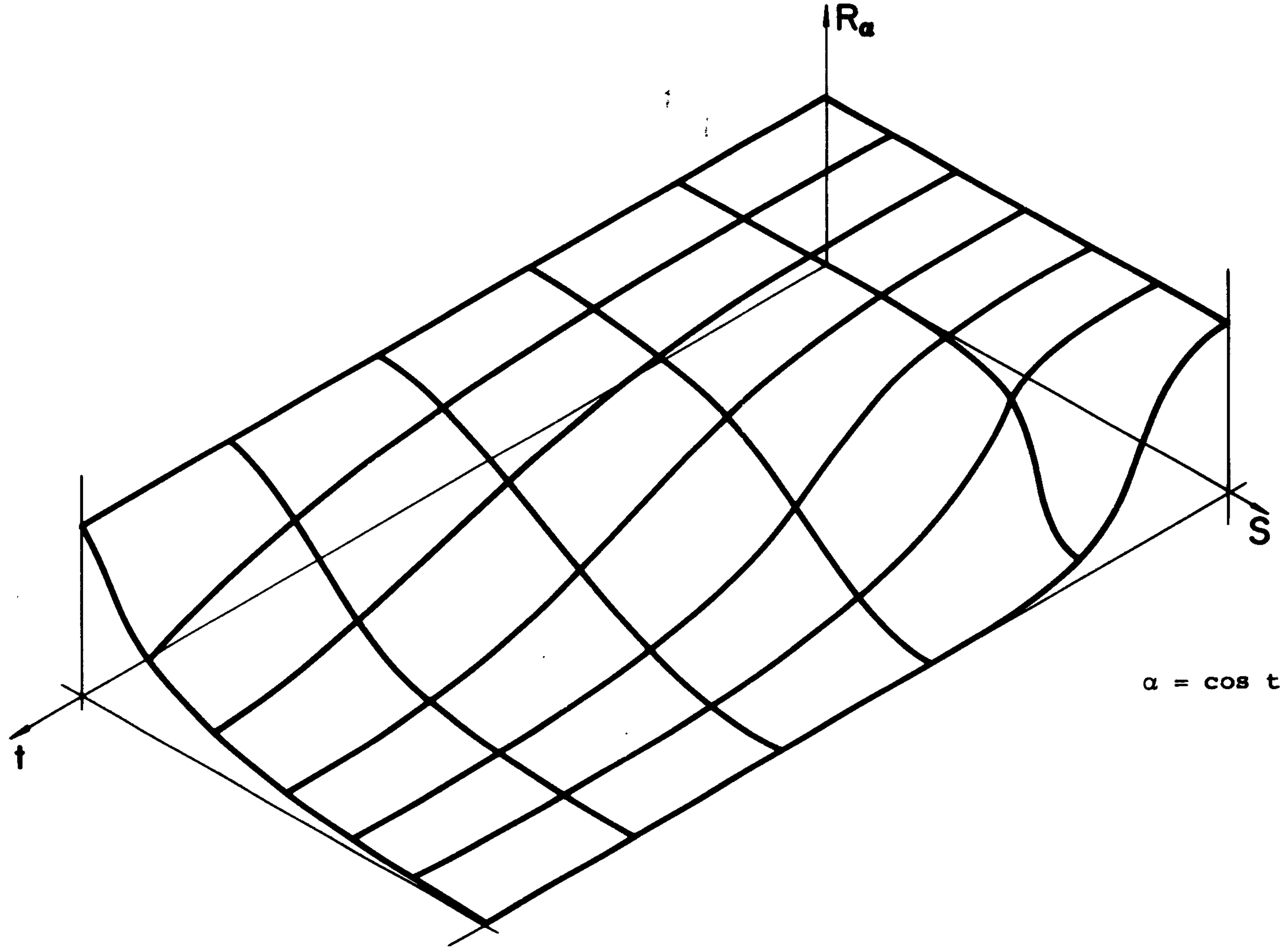
R_α
S
t
α = cos t

Fig. 8 Reliability Surface $R_\alpha(S,t)$ at Constant Angle
$\alpha = \arctan\,(S_m^\alpha/S_a)$

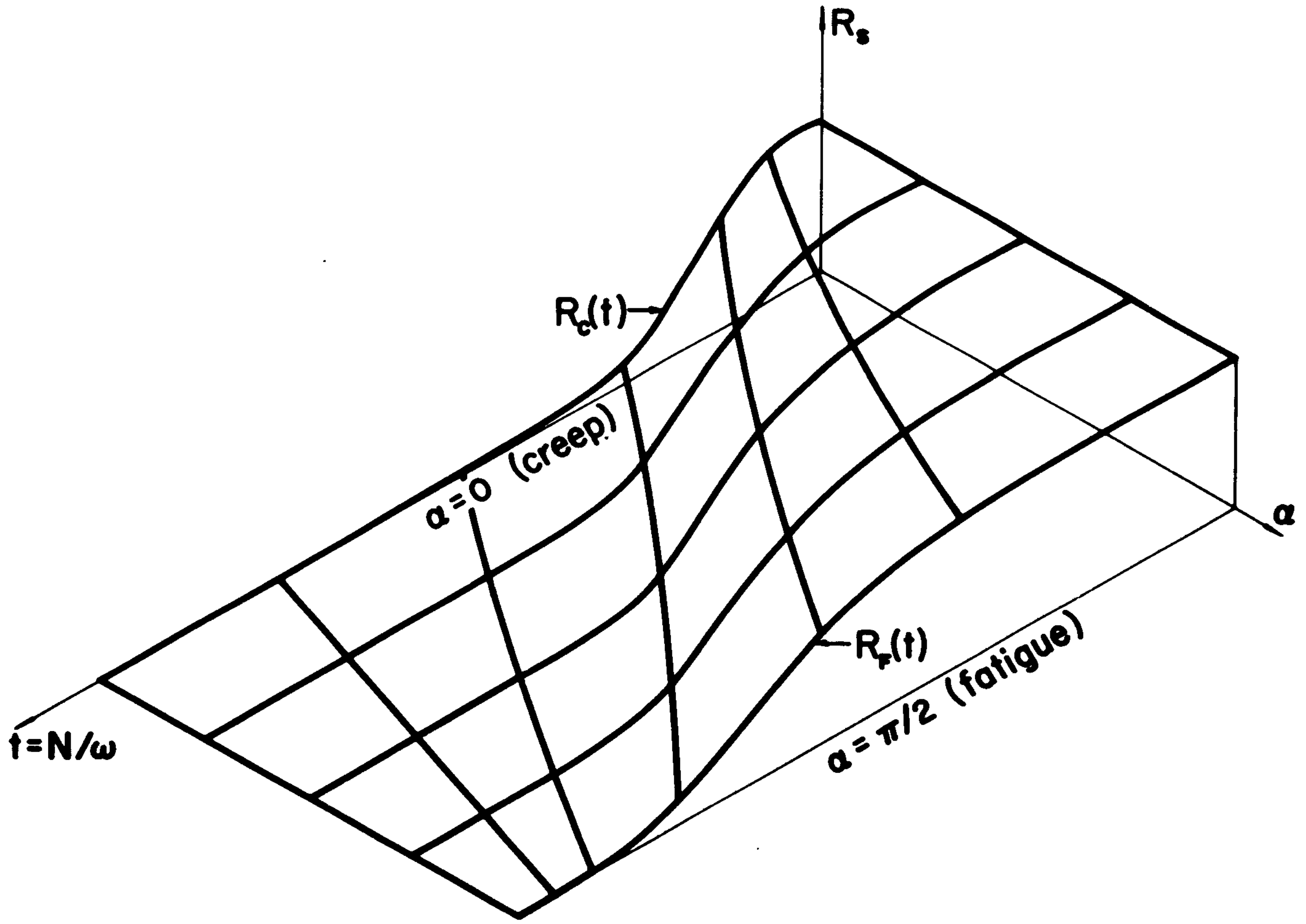
R_s
$R_c(t)$
$\alpha = 0$ (creep)
α
$R_F(t)$
$\alpha = \pi/2$ (fatigue)
$t = N/\omega$

Fig. 9 Reliability Surface $R_S(\alpha,t)$ at Constant Stress Intensity $S = \sqrt{S_m^2 + S_a^2}$

Aeronautical Systems Division, Dir/Materials and Processes, Metals and Ceramics Lab, Wright-Patterson AFB, Ohio.
Rpt Nr ASD-TDR-63-267 ASPECTS OF RELIABILITY UNDER CONDITIONS OF ELEVATED TEMPERATURE CREEP AND FATIGUE. Summary report, Mar 63, 39p. incl illus., tables, 12 refs.

Unclassified report

The solution of the problem of attaining adequate safety and reliability in supersonic aircraft structures operating under conditions under which the damaging effects of cycle sensitivity (fatigue) and time-sensitivity (creep) of the structural

(over)

1. Structural Reliability
2. Creep-Fatigue Interaction
I. AFSC Project 7351, Task 735106
II. Contract AF 33 (616)-6288
III. Ohio State Univ. Research Foundation, Columbus, Ohio
IV. A. M. Freudenthal
V. Aval fr OTS
VI. In ASTIA collection

material combined in gradually reducing the resistance of the structure requires the development of simplified procedures for the evaluation of the combined damage accumulation, which embody both the physical and probabilistic aspects of design.

The present report attempts to develop the basis for an approach to the solution of this problem, for which at present no adequate experimental information exists, and one of its purposes is to provide the guidelines for the planning of tests and experiments, the results of which would be relevant for structural design.

Aeronautical Systems Division, Dir/Materials and Processes, Metals and Ceramics Lab, Wright-Patterson AFB, Ohio.
Rpt Nr ASD-TDR-63-267 ASPECTS OF RELIABILITY UNDER CONDITIONS OF ELEVATED TEMPERATURE CREEP AND FATIGUE. Summary report, Mar 63, 39p. incl illus., tables, 12 refs.

Unclassified report

The solution of the problem of attaining adequate safety and reliability in supersonic aircraft structures operating under conditions under which the damaging effects of cycle sensitivity (fatigue) and time-sensitivity (creep) of the structural

(over)

1. Structural Reliability
2. Creep-Fatigue Interaction
I. AFSC Project 7351, Task 735106
II. Contract AF 33 (616)-6288
III. Ohio State Univ. Research Foundation, Columbus, Ohio
IV. A. M. Freudenthal
V. Aval fr OTS
VI. In ASTIA collection

material combined in gradually reducing the resistance of the structure requires the development of simplified procedures for the evaluation of the combined damage accumulation, which embody both the physical and probabilistic aspects of design.

The present report attempts to develop the basis for an approach to the solution of this problem, for which at present no adequate experimental information exists, and one of its purposes is to provide the guidelines for the planning of tests and experiments, the results of which would be relevant for structural design.